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Algorithm to Construct Super Vertex Magic Total Labeling of Complete Graphs

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ABSTRACT

The study of graph labeling has focused on finding classes of graphs which admits a particular type of labeling. In this paper we consider a particular class of graph which demonstrates Super Vertex Magic Total Labeling. The classes we considered here is a complete graph Kn. Various graph labeling that generalize the idea of a magic square have been discussed. In particular a vertex magic labeling on a graph with v vertices and e edges will be defined as a one to one map taking the vertices and edges onto the integers 1,2,3,………,v+e with the property that the sum of the label on a vertex and the labels of its incident edges is constant independent of the choice of vertex. The Super Vertex Magic Total Labeling of a graph is the Vertex Magic Labeling with the condition that all the vertices of the takes the labels 1,2,3,….,v. We use the magic square of order n to construct Super Vertex Magic Total Labeling for Kn.

Keywords: Magic squares, Magic constant, Complete graphs, Vertex Magic Total Labeling, Super Vertex Magic Total Labeling, etc.

1. INTRODUCTION

Let G = (V, E) be a graph which is finite, simple and undirected .The graph G has a vertex set V = V (G) and edge set E = E(G) .We denote an e = |E| and v = |V |. A standard graph theoretic notation is followed. In this paper we deal only with complete graphs. The labeling of a graph is a map that takes graph elements V or E or V U E to numbers (usually non-negative integers).In this paper, the domain is a set of all vertices and edges giving rise to total labeling. The most complete recent survey of graph labeling is [1].

Sedlacek introduced the magic labeling concept in 1963. The notion of an antimagic graph was introduced by Hartsfield and Ringel in 1989 [2]. Subsequently, as mentioned by Nicholas et al. [3], Bodendiek and Walther in 1996 [4] were the first to introduce the concept of (a,d)- vertex antimagic edge labeling, they called this labeling (a,d)-antimagic labeling. In both magic and antimagic labeling, we consider the sum of all labels associated with a graph element.

Formal definitions of magic labeling, vertex magic, edge magic, (a,d)-vertex antimagic total labeling, (a,d)-vertex antimagic edge labeling, vertex magic total labeling, edge magic total labeling and super vertex magic total labeling of graphs are as follows.

A valuation is the process of assigning integers to graph elements with some property to be satisfied is called labeling. If the domain consists of only vertex set V then it is called vertex magic, if the domain is the edge set then it is called edge magic, if the domain consists of both vertices and edges it leads to the total magic labeling.

A bijection f:E→{1,2,3,….,e} is called an (a,d)-vertex antimagic edge labeling, in short (a,d)-VAE labeling of G, if the set of vertex weights of all the vertices in G is {a,a+d,….,a+(v-1)d}, where a>0 and d≥0 are two fixed integers.

A bijection f: E U V→{1,2,3,…..,e+v} is called an (a,d)-vertex antimagic total lageling of G, if the set of vertex weights of all the vertices in G is {a,a+d,…..a+(v-1)d} where a>0 and d≥0 are two fixed integers. An (a,d)-vertex antimagic total lageling f is called a super (a,d)-vertex antimagic total lageling if f(V)={1,2,…….,v} and f(E)={v+1,……….,v+e}. If d=0 then (a,d)-VAT labeling is called a vertex magic total labeling (VMTL). This labeling was introduced by McDougall et al. [5] in 2002. Lin and miller [6] showed that all complete graphs Kn admits VMTL using mutually orthogonal latin squares. In [7,8] Krishnappa.H.K. and N.K.Srinath showed the same result by using magic square and divide and conquer method of algorithm design.

In 2004, K.A.Sugeng and others [9,10,11] introduced the notion of super vertex magic total labeling (SVMTL) and super edge magic total labeling (SEMTL) and they showed that all complete graphs Kn are SVMTL for n is odd.

In this paper we have proposed an algorithm to construct super vertex magic total labeling of complete graphs Kn where n is odd and n≡0 mod 4, where n>4. Also we proved that Kn, where n≡2 mod 4 does not have SVMTL.

2. CONSTRUCTION

The process of constructing SVMTL of complete graph is carried out as follows:

- K_n , where n is odd.
- K_n , where $n \equiv 0 \bmod 4$ and $n > 4$.
- K_n , where $n \equiv 2 \bmod 4$ are not SVMTL.

2.1 Theorem: The magic constant k for the given K_n lies within the range $(n(n^2+3))/4$ to $(n(n+1)^2)/4$.

Proof: The complete graph K_n has n vertices and $(n(n-1))/2$ edges. Therefore to label the graph elements we have to choose numbers from the set {1, 2, 3, …. (n(n-1))/2 + n}. If we choose {1, 2, 3, …. (n(n-1))/2} numbers to label the edges and (n(n-1))/2+1…(n(n-1))/2 + n to label the vertices, we will get the minimum magic constant $(n(n^2+3))/4$.

Let $m = \frac{n(n-1)}{2}$

$K_{min} = \frac{[2\sum_{i=1}^{m} i + \sum_{j=1}^{n}(m+j)]}{2}$

$2\sum_{i=1}^{m} i = 2[1+2+\ldots+m]$

$= 2[m(m+1)/2]$

Substitute for m=(n(n-1))/2 and upon simplification, we get

$2\sum_{i=1}^{m} i = \frac{n^4 - 2n^3 + 3n^2 - 2n}{4}$

Similarly, $\sum_{j=1}^{n}(m+j) = (m+1) + (m+2) + (m+3) + \ldots + (m+n)$

$= mn + \frac{n(n+1)}{2}$

$= \frac{n^2(n-1)}{2} + \frac{n(n+1)}{2}$

$= \frac{n^3+n}{2}$

So, $K_{min} = \frac{[\frac{n^4 - 2n^3 + 3n^2 - 2n}{4} + \frac{n^3+n}{2}]}{n}$

$= \frac{[\frac{n^4 - 2n^3 + 3n^2 - 2n + 2n^3 + 2n}{4}]}{n}$

$= \frac{n^4+3n^2}{4n} = \frac{n^3+3n}{4} = \frac{n(n^2+3)}{4}$

If we choose {1, 2,3, … n} to label the vertices of K_n and n+1, n+2, ….. + (n+n(n-1)/2) to label the edges of K_n, we will get the maximum magic constant.

$K_{max} = \frac{n(n+1)^2}{4}$

$= \frac{[\sum_{i=1}^{n} i + 2\sum_{j=1}^{m}(n+j)]}{n}$

$= \frac{[\frac{n(n+1)}{2} + 2(mn + \frac{m(m+1)}{2})]}{n}$

Substitute for m=(n(n-1))/2 and upon simplification, we get

$= \frac{[\frac{n(n+1)}{2} + 2(\frac{n^4 + 2n^3 - n^2 - 2n}{8})]}{n}$

$= \frac{[\frac{n^4 + 2n^3 - n^2 - 2n + 2n^2 + 2n}{4}]}{n}$

$$= \frac{n^4 + 2n^3 + n^2}{4n} = \frac{n(n+1)^2}{4}$$

2.2 Algorithm to construct SVMTL for Kn, where n is odd.

Algorithm **SVMTL_Kn(n)**
// This algorithm takes an integer parameter n, the number //of vertices of the complete graph Kn. This algorithm //construct an n x n matrix, the construction is similar to //the construction of magic square of order n. Let M //denotes such a n x n matrix and i, j are row and column //indices respectively. The array S contains numbers from //1 to n+m, where m=n(n-1)/2.

Step-1: [Initialization]
i←1; j←1; m←n(n-1)/2;
Repeat for k←1 to n+m do
S[k]←k;
Step-2: Fill the matrix M starting with [i, j] in such a way that the filling process proceeds in down-right direction using the numbers of S. Assume that the rows and columns are cyclically around. If the present position is already filled then get back to the previous position and start filling from its down position. Repeat this until all the numbers from S are filled in M.
Step-3: At this stage n+n(n-1)/2 cells of M are filled and the remaining n(n-1)/2 cells are empty. Fill these empty cells by symmetry i.e M[i,j]=M[j,i].
Step-4: Now use the entries of M to label the vertices and edges of Kn as follows. The entries across the diagonal are used to label the vertices and the left out entries to label the edges of the complete graph Kn.
Step-5: Stop.

Applying this algorithm to K_3 and K_5 are illustrated bellow.
Example 1: K_3 Here n=3 and m=3(3-1)/2=3 and so S= {1, 2, 3, 4, 5, 6}.

1		

1		
	2	

1		
	2	
		3

1		
	2	4
		3

1		
	2	4
5		3

1	6	
	2	4
5		3

By symmetry fill the remaining cells.

Table 1: VMTL of K_3

1	6	5
6	2	4
5	4	3

The magic constant is 12= $[3(3+1)^2]/4$.
Mapping of the matrix entries to vertices and edges of K_3 is as shown in fig (1) below.

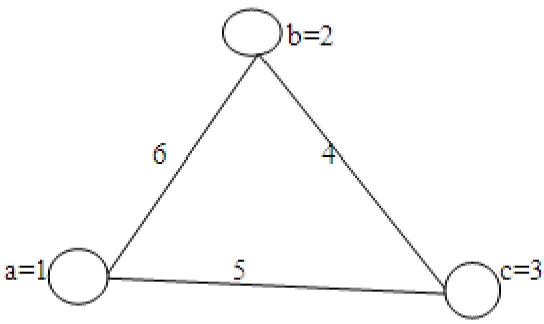

Fig 1: Labeled graph K_3

Example 2: K_5 Here n=5 and m=5(5-1)/2=10 and so
S= {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15}.

Table 2: VMTL of K_5

1				
	2			
		3		
			4	
				5

1			10	
	2			6
7		3		
	8		4	
		9		5

1	14		10	
	2	15		6
7		3	11	
	8		4	12
13		9		5

1	14	7	10	13
14	2	15	8	6
7	15	3	11	9
10	8	11	4	12
13	6	9	12	5

The magic constant is 45= $[5(5+1)^2]/4$.
Mapping of the matrix entries to vertices and edges of K_5 is as shown in fig (2) below.

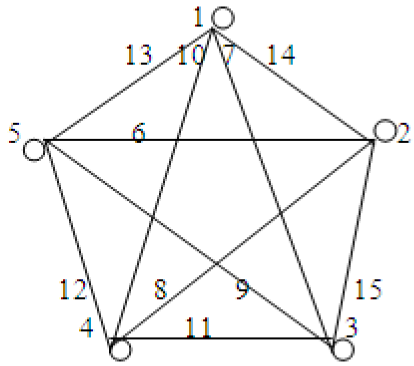

Fig 2: Labeled graph K_5

2.3 There exists no SVMTL for K_n , where n ≡ 2 mod 4.

From theorem 2.1 it is known that the maximum magic constant for K_n is $[n(n+1)^2]/4$ for any positive integer n. If n is even and n ≡ 2 mod 4, the equation $[n(n+1)^2]/4$ evaluates to a non integer, hence it is not possible to obtain SVMTL for K_n , where n ≡ 2 mod 4.

Example 3: if n=6, then $[n(n+1)^2]/4$ becomes

$[6(6+1)^2]/4 = (6 \times 49)/4 = 73.5$.

2.4 There exists SVMTL for K_n , where n ≡ 0 mod 4.

In [12] it is known that it is possible to realize all possible magic constants for Kn. From theorem 2.1 it is known that the maximum magic constant for K_n is $[n(n+1)^2]/4$ for any positive integer n. If n is even and n ≡ 0 mod 4, the equation $[n(n+1)^2]/4$ evaluates to an integer, hence it is possible to obtain SVMTL for K_n , where n ≡ 0 mod 4.

Example 4: For K_8, here n=8 and the magic constant is $[8(8+1)^2]/4 = 162$.
Using the algorithm in [12], the 8 x 8 matrix which realizes the SVMTL for K_8 is as shown below.

Table 3: VMTL of K_8

-	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
1	-	15	19	16	24	35	31	21
2	15	-	23	11	12	30	33	36
3	19	23	-	20	27	10	28	32
4	16	11	20	-	34	29	22	26
5	24	12	27	34	-	25	18	17
6	35	30	10	29	25	-	14	13
7	31	33	28	22	18	14	-	9
8	21	36	32	26	17	13	9	-

k	**162**	**162**	**162**	**162**	**162**	**162**	**162**	**162**

3. CONCLUSION

From section2.1, section2.2, and section2.3 it follows that all complete graphs Kn, for n>2 there is a super vertex magic total labeling (SVMTL) except for K4 and Kn , where n ≡ 2 mod 4. .

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