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PROPOSED METHOD FOR SOLVING FTSP

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Abstract

The traveling salesman problem (TSP) is well known optimization problem. The goal of the traveling salesman (TS) has to visit 'n' cities. He needs to start from a particular city visit each city once and then return to his starting point. His aim is to select the sequence in which the cities are visited in such a way that his total travelling time is minimized. We attempt in this paper to lead a new approach to fuzzy traveling salesman problem (FTSP). In this paper we use discrete fuzzy number to signify the vagueness. Discrete fuzzy numbers are then transformed to the triangular fuzzy number and trapezoidal fuzzy number (TFNs). The TFNs are defuzzificated by yager's ranking technique and a new algorithm is proposed to solve FTSP. In this way to get noble solutions in the shorter time in the case that objective function, this technique is a regular procedure, easy to apply and can be consumed for all types of FTSP with minimize the objective function. At the end, this method is explained with suitable numerical examples.

Keywords: Traveling salesman problem, Optimization technique, Triangular and Trapezoidal fuzzy number, Defuzzification, Yager's ranking technique.

Mathematics Subject Classification : 90C05, 90C10, 90C70

1. INTRODUCTION

The traveling salesman problem (TSP) first proposed by Irish mathematician W.R Hamilton in the 19th century [12]. A large number of techniques were developed to solve the problem. The objective or goal of the problem is to invent the shortest route of the salesman starting from a given city, visiting all other cities only once and finally come to the same city where he started. This problem is known to be NP (non -deterministic polynomial time) complete and cannot be solved exactly in polynomial time because the number of possible routes increases factorally with the number of cities. In other words, for 4 cities there are $4!$ (24) possible routes and for 5 cities there are $5!$ (120). A salesman who had to visit the 20 cities in this example would have 2432290….possible routes to consider. This traditional approach become impossible in terms of compute memory and speed constraints. So to solve this problem different researchers use heuristic, mata heuristic and optimal method like dynamic programing, linear programming[2], branch and bound method, Tabu such method, cutting plane algorithms, simulated annealing, Markov chain. Few more algorithms like particle swarm optimization and system, evolutionary computation, artificial colony, neural networks etc., also there

In real life simulation it may not be possible to get the cost or time as certain quantity. To overcome this Zadeh[13] lead fuzzy set concepts to deal with in accuracy and vagueness. Since then major advantages have been made in-developing numerous methods and their applications to several decision problem. If the cost or time or distance is not crisp values, then it becomes a TFNs . As Voxman [10] first introduced the concept of direct fuzzy number, we translated discrete fuzzy number in to TFNs. Travel time with TFNs make. The objective functions of TSP nonlinear, but since nonlinear models cannot guarantee the optimal solution, we relax the NLP by converting the model to LP finds solution with a little deterioration. In recent years Fuzzy TSP(FTSP) has got great attention and the problems in FTSP have been approached using several technique. In paraqueta et al[6], the extended local search algorithm called pareto local search algorithm was introduced. In the work of Mukerjee and Basu[4], a new method was proposed to solve FTSP. Tavakoli Moghaddan.R et.al [9] used fuzzy multi objectives linear programming to solve multi objective single machine scheduling problem. Liang et al [3] applied fuzzy multi objective linear programming for distribution planning decision Sepidch Fereidouni[7] approached the problem using multi objective linear programming.

In this paper, a new algorithm which is similar to classical assignment method is introduced to solve FTSP. Thus, In this paper it is organized as follows. In section 2, some of the preliminary concepts on fuzzy number and function principle are given. In section 3 formulation of Traveling salesman problem. In section 4, formulation of Traveling sales man problem as an assignment problem. In section 5 and in section 6, defuzzyed the TFNs through Yager's ranking technique. In section 7, the proposed algorithm is discussed. In section 8, the numerical example is given and in section 9 the paper is concluded.

2. PRELIMINARIES OF FUZZY SETS

In 1965, Zadeh [13] introduced the concept of fuzzy set as a mathematical way of representing impreciseness in real world problems

2.1 Fuzzy set

A fuzzy set is characterized by a membership function mapping element of a domain, space, or the universe of discourse X to the unit interval [0, 1] i.e. $A=\{(x,\mu_A(x));x\in X\}$. Here $\mu_A : X\rightarrow[0,1]$ is a mapping called the degree of membership function of the fuzzy set A and $\mu_A(x)$ is called the membership value of $x\in X$ in the fuzzy set A. These membership grades are often represented by real numbers ranging from [0, 1].

2.2 Triangular and Trapezoidal fuzzy number

A fuzzy number A is a triangular fuzzy number denoted by A (a,b,c) with membership function $\mu_A(x)$ given by

$$\mu_A(x)=\begin{cases}\frac{x-a}{b-a}, & a\le x\le b\\ 1 \quad , & x=b\\ \frac{c-x}{c-b}, & \text{b}\le x\le c\\ 0 & ,otherwise\end{cases}$$

and

A fuzzy number A is a trapezoidal fuzzy number denoted by A (a,b,c,d) with membership function $\mu_A(x)$ given by

$$\mu_A(x)=\begin{cases}\frac{x-a}{b-a}, & a\le x\le b\\ 1 \quad , & \text{b}\le x\le c\\ \frac{d-x}{d-c}, & \text{c}\le x\le d\\ 0 & ,otherwise\end{cases}$$

2.3 α-cut

The α-cut of a fuzzy number $A(x)$ is defined as $A(\alpha)=\{x/\mu(x)\ge\alpha,\alpha\in[0,1]\}$

2.4 Arithmetic Operation of two TFNs

$$\tilde{A}+\tilde{B}=(a_1+b_1,a_2+b_2,a_3+b_3)$$

$$\tilde{A}-\tilde{B}=(a_1-b_3,a_2-b_2,a_3-b_1)$$

and

$$\tilde{A}+\tilde{B}=(a_1+b_1,a_2+b_2,a_3+b_3,a_4+b_4)$$

$$\tilde{A}-\tilde{B}=(a_1-b_4,a_2-b_3,a_3-b_2,a_4-b_1)$$

3. FORMULATION OF TRAVELING SALESMEN PROBLEM

The problem whose solution will yield the minimum travelling time is , let the variables x_{ij} be defined as

$$x_{ij}=\begin{cases}1, & \text{form city i to city j}\\ 0, & \text{other wise}\end{cases}$$

Thus, the above model can be expressed as

$$Minimize\ \tilde{Z}=\sum_i\sum_j\tilde{C}_{ij}x_{ij}$$

Subject to the constraints

$$\sum_{i=1}^{m}x_{ij}=1 \quad \text{j=1,2,..,n}$$

$$\sum_{j=1}^{n}x_{ij}=1 \quad \text{i=1,2,..,m}$$

$$x_{ij}=0\ or\ 1$$

4. Formulation Of Traveling Salesman Problem As An Assignment Problem

A travelling salesman (TS) has to visit 'n' cities and return to the starting city. He has to start from any one city and visit each city only once. Suppose he starts from the kth city and the last city he visited is m. let the cost of travel form ith city to jth city be c_{ij}.Then the objective function is

$$Minimize\ \tilde{Z}=\sum_{i}^{m}\sum_{j}^{n}\tilde{C}_{ij}x_{ij}$$

Subject to the constraints

$$\sum_{i=1}^{m}x_{ij}=1,\quad j=1,2,..,n,\ i\neq j, i\neq m$$

$$\sum_{j=1}^{n}x_{ij}=1,\quad i=1,2,..,m,\ i\neq j, i\neq m$$

$$x_{mk}=1$$

$$x_{ij}=0\ or\ 1$$

5. Defuzzification

Defuzzification[5] is the process of finding the singleton value(crisp value) which represents the average value of the TFNs. There is no unique way to perform the defuzzification. The several existing methods for defuzzificationis commonly accepted in the literature. In this paper, we use Yager's ranking to defuzzify the TFNs because of its simplicity and accuracy.

6. Yager's Ranking Technique

Yager's ranking technique [11] which satisfy compensation, linearity, and additively properties and provides results which are consist of human intuition. If $\tilde{a}$ is a fuzzy number then the Yager's Ranking is defined by

$R(\tilde{a})=\int_{0}^{1}0.5(a_l,a_u)\,d\alpha$ Where (a_l,a_u) is the α-level cut of the fuzzy number $\tilde{a}$

7. The Proposed Method For Fuzzy Traveling Salesman Problem

Step1. In a minimization case, find the minimum element of each row in the assignment matrix(say a_i) and write it on the right hand side of the matrix .Then divide each element of i^{th} row of the matrix by a_i.These operations create at least one ones in each rows. In term of ones for each row and column do assignment, otherwise go to step 2.

Step2. Find the minimum element of each column in assignment matrix (b_j), and write it below j^{th} column. Then divide each element of j^{th} column of the matrix by b_j.These operations create at least one ones in each columns. Make assignment in terms of ones. If no feasible assignment can be achieved from step(1) and (2) then go to step3.

Step 3. Draw the minimum number of lines to cover all the ones of the matrix. If the number of drawn lines less than n, then the complete assignment is not possible, while if the number of lines is exactly equal to n, then the complete assignment is obtained.

Step 4. If a complete assignment program is not possible in step 3, then select the smallest element (say d_{ij}) out of those which do not lie on any of the lines in the above matrix. Then divide by d_{ij} each element of the uncovered rows or columns, which d_{ij} lies on it. This operation creates some new ones to this row or column. If still a complete optimal assignment is not achieved in this new matrix, then use step 4 and 3 iteratively. By repeating the same procedure the optimal assignment will be obtained [1].

Priority plays an important role in this method, when we want to assign the ones.

Priority rule. For minimization assignment problem, assign the ones on the rows which have smallest element on the right hand side, respectively.

Step 5. If Solution obtained contains a path which starts from given city and covers all the other cities exactly once and terminates again at starting city then, the optimal solution travelling salesman problem is obtained. Otherwise go to step 6.

Step 6 . After solving the given problem by fuzzy assignment technique, use the method enumeration by assigning in a cell(say A) having minimum rank other then one of the matrix instead of assigning in cell having one rank. Cut the column corresponding to this assignment. The remaining assignments can be made according to the same fuzzy assignment technique [8].

Step 7. Repeat step 6 until, the closed path is not obtained.

8. NUMERICAL EXAMPLE

The proposed method has been explained through the example given below:

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{pmatrix} \infty & (4,6,8,10) & (5,7,9,11) & (6,8,10,12) \\ (4,6,8,10) & \infty & (2,4,6,8) & (1,3,5,7) \\ (5,7,9,11) & (2,4,6,8) & \infty & (3,5,7,9) \\ (6,8,10,12) & (1,3,5,7) & (3,5,7,9) & \infty \end{pmatrix} \end{array}$$

Now we calculate R(4,6,8,10) by applying Yager's ranking method. The membership function of the triangular fuzzy number (4,6,8,10) is

$$\mu(x)=\begin{cases} \dfrac{x-4}{2}, & 3 \le x \le 5 \\ 1 \quad , & 6 \le x \le 8 \\ \dfrac{10-x}{2}, & 8 \le x \le 10 \\ 0 & ,otherwise \end{cases}$$

The α-Cut of the fuzzy number (4,6,8,10) is $\left(a_\alpha^l, a_\alpha^u\right)=(2\alpha+4, 10-2\alpha)$ for which $R\left(\tilde{a}_{1,2}\right)=R(4,6,8,10)$

$$=\int_0^1 0.5\left(a_\alpha^l, a_\alpha^u\right)d\alpha=\int_0^1 0.5(14)d\alpha=7$$

Proceeding similarly, the Yager's ranking indices for the fuzzy costs a_{ij} are calculated as: $R\left(\tilde{a}_{1,3}\right)$=8, $R\left(\tilde{a}_{1,4}\right)$=9, $R\left(\tilde{a}_{2,1}\right)$=7, $R\left(\tilde{a}_{2,3}\right)$=5, $R\left(\tilde{a}_{2,4}\right)$=4, $R\left(\tilde{a}_{3,1}\right)$=8, $R\left(\tilde{a}_{3,2}\right)$=5, $R\left(\tilde{a}_{3,4}\right)$=6, $R\left(\tilde{a}_{4,1}\right)$=9, $R\left(\tilde{a}_{4,2}\right)$=4,$R\left(\tilde{a}_{4,3}\right)$=6

We replace these values for their corresponding $\tilde{a}_{ij}$ in which result in a convenient assignment problem in the linear programming problem.

$$=\begin{pmatrix} \infty & 7 & 8 & 9 \\ 7 & \infty & 5 & 4 \\ 8 & 5 & \infty & 6 \\ 9 & 4 & 6 & \infty \end{pmatrix}$$

By proposed Method, we get

$$=\begin{pmatrix} \infty & 7 & 8^* & 9 \\ 7 & \infty & 5 & 4^* \\ 8^* & 5 & \infty & 6 \\ 9 & 4^* & 6 & \infty \end{pmatrix}$$

Therefore, the total travel cost is $R(5,7,9,11)+R(1,3,5,7)+R(5,7,9,11)+R(1,3,5,7)=R(12,20,28,36)$ = 8+4+8+4=24

9. CONCLUSION

In this paper, a new algorithm has been proposed to solve the fuzzy travelling salesman problems occurring in real life situation. To illustrate the algorithm a numerical example has been solved in which approximate cost is represented as a TFNs. If there is no uncertainty about the cost i.e. if the cost is not fuzzy then the proposed algorithm gives the same result as in crisp travelling salesman problem.

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