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Application of Centred Scheme for capturing material interface in Compressible Multifluids

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Abstract

This paper is concerned with the application of Centred numerical scheme for compressible multicomponent fluids. The fluid components are assumed to be immiscible and are separated by material interfaces. In general, compressible Euler equations consist of conservation of mass, momentum and energy. Here one additional equation is considered as conservation law for thermodynamic state variable, specific heat ratio (γ). The First ORder CEntred (FORCE) finite volume scheme is implemented to capture the material interface in multifluid with different values of γ. Numerical experiments are carried out for test cases of shock tube and stiff shock tube problem. The results are demonstrated for the feasibility of the FORCE method in capturing the interface in single fluid flow.

Keywords: Compressible Euler Equations, Material Interface, Shock Tube, FORCE Centred Scheme

1. Introduction

In recent years the interest is growing into understanding the dynamics of fluids consisting of several fluid components .including the study of dynamics of interfaces in flow, bubbly flows and oil recover. Dynamics of these flows are challenging problems for both theoretically as well as numerically. In this work we have concentrated on the single fluid flow problem.

It is assumed that the fluid components are described by a single velocity and a single pressure, the flow is described by the compressible Euler equations such as

$$(\varrho)_t+(\varrho u)_x=0,$$
$$(\varrho u)_t+(\varrho u^2+p)_x=0,$$
$$(E)_t+(u(E+p))_x=0. \quad (1)$$

The system (1) depict the conservation of mass, momentum, and energy of the fluid mixture. The notations ϱ, u and p are the density, velocity and pressure of the fluid, respectively. The total energy (E) is related to internal energy (e) as

$$E=\varrho e+\frac{\varrho u^2}{2}. \quad (2)$$

The fluid components are separated by material interfaces, and corresponding equation of state (EOS) are different for different components. The EOS for stiff gases is given by:

$$\varrho e=\frac{p+\gamma p_\infty}{\gamma-1}, \quad (3)$$

where e denotes the specific internal energy, γ the ratio of specific heats of the fluid component, and p_∞ its stiffness parameter, with p_∞=0 corresponding to the ideal gas case. The speed of sound is given as

$$c=\sqrt{\frac{\gamma(p+p_\infty)}{\varrho}}. \quad (4)$$

Here we have assumed that the fluid consists of two components and are identified by variable ψ. The Various choices of ψ have been suggested in the literature [1], [2], depending on the model assumptions. We have considered the state variable specific heat ratio γ [2] for variable ψ, which propagates with the fluid velocity, hence satisfies the equation:

$$\psi_t+(u\psi)_x=0, \tag{5}$$

which may be combined with the first equation of system (1) and its conservation form is

$$(\varrho\psi)_t+(\varrho u\psi)_x=0. \tag{6}$$

There are several numerical methods [3, 4] have been proposed from different perspectives to treat material interfaces. The material interfaces are considered as genuine discontinuity and treated them as material boundaries in Arbitrary Lagrangian Eulerian methods [6] and free-Lagrange methods [5], [10]. Glimm et. al. [7], [8] discussed about the advantage of both the Eulerian and Lagrangian approaches for tracking material front. In this work they have treated the interface as one of the moving internal boundary. These methods are good in capturing interfaces as sharp discontinuities but their extensions in multiple space dimensions are difficult.
In this paper, we focus the application of centred numerical method for multimaterial flows consisting of pure fluids separated by material interfaces. Here we have applied a First ORder CEntred (FORCE) scheme developed by Toro [9] for capturing the material interface in multifluids in one dimesion. The FORCE scheme is monotonic and depends on the left and right state values of flow variables. This scheme is simple and easy to implement, provided governing equations are written in conservative form.
The paper is organized as follows. In Section 2, a brief review of FORCE scheme is given. The numerical experiments for shock tube problems using FORCE scheme for capturing material interface is demonstrated in section 3. Conclusions are stated in Section 4.

2. Central scheme

The compressible Euler equations stated in (1) can be written as

$$\frac{\partial Q(x,t)}{\partial t}+\frac{\partial F(Q(x,t))}{\partial x}=0 \tag{7}$$

where $Q(x,t)$ is the vector of the conserved variables and $F(U)$ is the corresponding vector of flux. The initial conditions are given as

$$Q(x,0)=Q^{(0)}(x),\ x\in R. \tag{8}$$

The finite volume numerical scheme can be written in the form of

$$Q_i^{n+1}=Q_i^n-\frac{\Delta t}{\Delta x}(F_{i+\frac{1}{2}}-F_{i-\frac{1}{2}}), \tag{9}$$

Here, we are discussing the FORCE flux introduced by Toro [9], which is the average of Lax Wendroff and Lax-Friedrichs fluxes. FORCE flux is given by

$$F_{i+\frac{1}{2}}^{FORCE}(Q_i,Q_{i+1})=\frac{1}{2}[F_{i+\frac{1}{2}}^{LW}(Q_i,Q_{i+1})+F_{i+\frac{1}{2}}^{LF}(Q_i,Q_{i+1})]. \tag{10}$$

The centred Lax-Friedrich flux is defined by

$$F_{i+\frac{1}{2}}^{LF}(Q_i,Q_{i+1})=\frac{1}{2}[F(Q_i)+F(Q_{i+1})]-\frac{\Delta x}{2\Delta t}[Q_{i+1}-Q_i], \tag{11}$$

and the two-step Lax-Wendroff flux is defined as

$$F_{i+\frac{1}{2}}^{LW}(Q_i,Q_{i+1})=F(Q_{i+\frac{1}{2}}^{LW}) \tag{12}$$

and

$$Q_{i+\frac{1}{2}}^{LW}(Q_i,Q_{i+1})=\frac{1}{2}[Q_{i+1}+Q_i]-\frac{\Delta t}{2\Delta x}[F(Q_{i+1})-F(Q_i)]. \tag{13}$$

The time step Δt has been chosen with Courant-Friedrichs-Lewy (CFL) restriction defined as

$$\Delta t=CFL\frac{\Delta x}{max(|u|+c)}, \tag{14}$$

where 0<*CFL*≤1. The approximate solution of (7) is obtained by substituting the numerical flux (10) into (9). The intercell numerical fluxes are estimated using (10). The transmissive boundary conditions are applied at the left and right boundary of the domain.

3. Simulation study

In this section, we illustrate the performance of the interface tracking method for two one dimensional test cases. In both the cases we use the First ORder CEntred (FORCE) scheme. To obtain the solution in the vicinity of material interface, we use approximate Riemann solver.

3.1 Two fluid Shock tube Problem

In this example we have explore a single gas phase with two values of γ. The domain is considered to be [0,1] and the interface is located midway at 0.5. The initial conditions [12] correspond to shock tube data for the left state and for right state are summarized in the table 1. We compute the solution of this problem at time *t*=0.2 on a uniform grid.

Table 1: Initial conditions for shock tube

Variables	Left state	Right state
Density (ϱ)	1.0	0.125
Pressure (p)	1.0	0.1
Velocity (u)	0.0	0.0
Ratio of specific heat (γ)	1.4	1.6

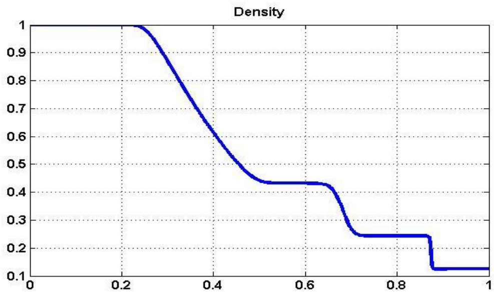

Figure 1: Solution (Density) of the shock tube problem by FORCE method

The simulation results are demonstrated for the flow parameters like pressure, density and velocity with space. In the Fig. 1, density variation has been plotted. It is seen that the interface is captured nicely since there are two values of γ, there are no oscillations near the interface. The sharp contact discontinuity is observed in the plot of velocity, shown in Fig.2. The expansion fan is also captured which can be seen in the pressure plot Fig.3.

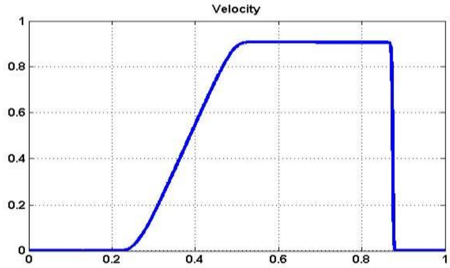

Figure 2: Velocity of the shock tube problem

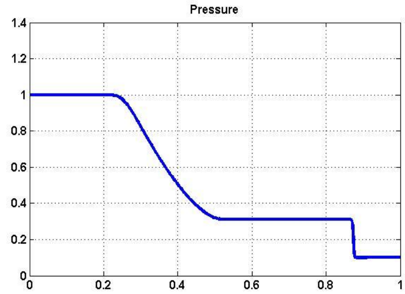

Figure 3: Pressure of the shock tube problem

3.2 Two fluid Stiff Shock tube Problem

Second numerical experiments are done for stiff shock tube problem, considered from [11] and its initial left state and right state values are summarized in the table 2. The computational domain [0,1] is discretized into uniform mesh and the initial position of interface is at *x*=0.5. The simulations are carried out for time *t*=0.015.

Table 2: Initial conditions for shock tube

Variables	Left state	Right state
Density (ϱ)	1.0	0.125
Pressure (*p*)	500.0	0.20
Velocity (*u*)	0.0	0.0
Ratio of specific heat(γ)	1.4	1.6

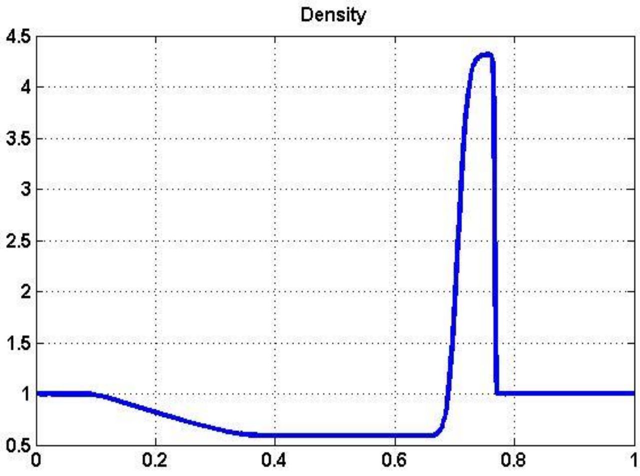

Figure 4: Density of the stiff shock tube problem

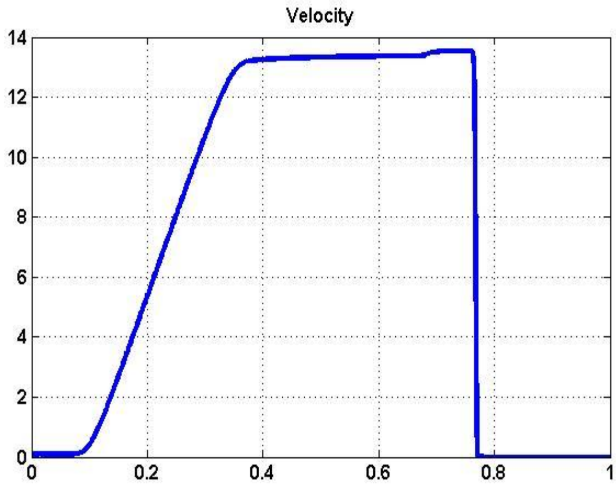

Figure 5: Velocity of the stiff shock tube problem

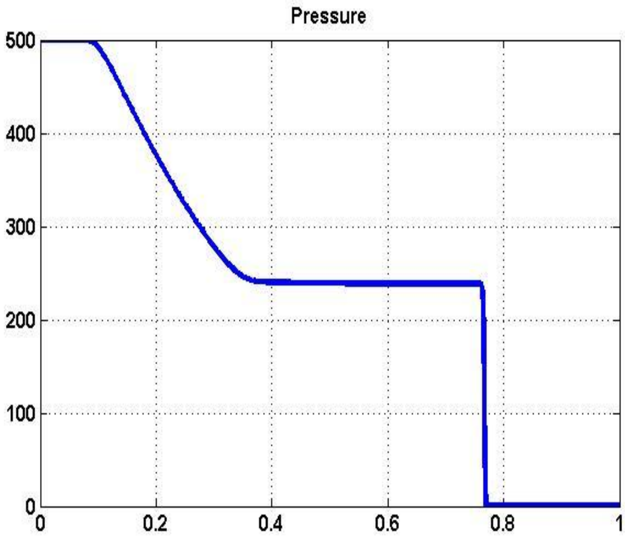

Figure 6: Pressure of the stiff shock tube problem

The solutions for density, velocity and pressure are plotted in Fig. 4-6. In this simulation, we have seen that the solutions are showing better sharpness for all the discontinuities present in the solution. Also we note that all numerical solutions are oscillation free.

4 . Conclusions

We have implemented First Order CEntred (FORCE) scheme for compressible multicomponent fluid flow governed by Euler equations for capturing material interface. The FORCE scheme is very simple and does not require the information of eigenstructure of the system. Numerical results presented in the paper show the feasibility of the scheme for tracking material interfaces in one dimension. The scheme can be studied for gas-liquid multifluid system in one dimensionon.

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